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Effect of initial stress on the propagation and attenuation characteristics of Rayleigh waves

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Abstract The present investigation deals with the mathematical modelling and analytical thinking to uncover the various facets of the propagation of Rayleigh waves in an Earth's crustal layer. This work has been carried out when the wave is passing through a pre-stressed anisotropic layer of finite thickness, lying over a semi-infinite medium with void pores. The upper boundary plane of the crustal layer has been thought to be a free surface. Displacement components of the wave for both the media have been derived analytically. Appropriate boundary conditions have been well satisfied with the aid of displacement and stress factors in order to get the desired dispersion relation. A comparative study has been performed graphically taking anisotropic, orthotropic and isotropic strata, in order to show the impact of initial stress and thickness on the propagation characteristics of Rayleigh waves. The present work may establish a program to connect theoretical results with subject area applications.

1 Introduction

Ever since the existence of Rayleigh waves was predicted by the British physicist Lord Rayleigh [1], surface waves have been extensively devoted as a dynamic observation paraphernalia in many disciplines, such as solid physics, seismology, geophysics, geotechnical engineering, and many more. The prime motive of all applications in these orbits is to find out media or material properties inside a certain range near the surface where surface waves propagate. Rayleigh waves are very much helpful not just in terms of characterization of materials, but also to uncover the mechanical and structural properties of the object being examined. Rayleigh waves are those surface acoustic waves that travel on solids and are part of the seismic waves that are brought forth in the Earth by earthquakes. They are likewise known as Lamb waves, generalized Rayleigh waves or Rayleigh–Lamb waves, when guided in layers. Rayleigh waves produced during earthquakes possessing low

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frequency are widely applied to characterize the Earth's interior, whereas those in the medium-frequency range are beneficial for the delineation of oil deposits.

Remarkable amount of data about the propagation of elastic waves in layered media have been extensively summarized in the books of Ewing et al. [2], Brekhovskikh [3], and Kennet [4]. The subject of seismic wave propagation has been extended to layered anisotropic media by Nayfeh [5]. Furthermore, the analysis of Rayleigh waves has been recorded remarkably in several treatises, including Achenbach [6] and Pilant [7]. In the year 1985, Bullen and Bolt [8] introduced the theory of seismology giving detailed information about seismic waves and their generation in different media possessing different characteristics.

Anisotropy is a directionally dependent property of a medium which briefly means varying properties along varying directions. Particularly, the dependence of seismic wave velocity on direction or upon angle is known as seismic anisotropy. In the Earth's crust, mantle, and inner core, significant seismic anisotropy has been discovered. Important facts and information regarding processes and mineralogy in the Earth can be easily assessed by measuring the effects of anisotropy in seismic data. Due to these applications in various fields, wave propagation in anisotropic media has constantly been a cynosure to researchers, scientists, and mathematicians. Synge [9] illustrated the propagation of elastic waves in anisotropic media, whereas Dutta [10] studied the Rayleigh wave propagation considering two layered anisotropic media. Furthermore, Sharma and Gogna [11] contributed their ideas on wave propagation in anisotropic liquid-saturated porous solids. A worldwide problem of elastic wave propagation taking multilayered anisotropic media into account has been remarkably discussed by Nayfeh [12]. Moreover, Vinh and Hue [13] demonstrated the impact of impedance boundary conditions in anisotropic solids on the propagation of Rayleigh waves. In the late years, the dispersion of Rayleigh waves in weakly anisotropic media with vertically inhomogeneous initial stress has been investigated by Tanuma et al. [14]. On the other hand, Pal et al. [15] briefly examined the propagation of Rayleigh waves in an anisotropic layer overlying a semi-infinite sandy medium.

Initial stress can be easily delineated as a particular stress, which persists in an elastic consistency, although external forces are unavailable, and hence, the body is said to be initially stressed. Moreover, initial stresses can be engendered in the medium because of both natural and artificial procedures. The evolution of initial stress is due to various reasons, for example, the variation of temperature, presence of overburdened layer, gravity variation, atmospheric pressure, and many more. These stresses possess pronounced impact not only on the propagation of seismic waves, but also on the stability of the medium. Earth may be looked at as a stratified medium under initial stress. Hence, it is of vast area of interest to examine the issue of these stresses on the propagation of Rayleigh waves. The governing equations of motion for a pre-stressed elastic medium have been introduced by Biot [16] in order to find out the impact of initial stresses on the propagation of elastic waves. It clearly elucidates that there exists a vast difference between the wave propagation under initial stress and the stress-free case. Chattopadhyay et al. [17] gave a brief mathematical overview on the propagation of Rayleigh waves in a pre-stressed medium. An extensive discussion regarding the reflection phenomenon of qP and qSV waves in a pre-stressed piezoelectric half-space has been provided by Singh [18]. Abd-Alla et al. [19] studied the traversal characteristics of Rayleigh waves in a generalized magneto-thermoelastic orthotropic material under initial stress and gravity field. Furthermore, Sharma and Gupta [20] analysed the impact of initial stress on the characteristics of Rayleigh wave propagation. The subject area of Rayleigh waves propagating in a magneto-electro-elastic half-space with initial stress has been discussed remarkably by Zhang et al. [21]. The influence of initial stress is prominent not only on Rayleigh waves, but also on other seismic waves as well. Recently, Pandit et al. [22] illustrated the effect of initial stress on the propagation of Love waves in a Voigt-type viscoelastic orthotropic functionally graded layer lying over a porous half-space.

A number of problems connected with the wave propagation in a medium containing void or vacuous pores have attracted eminent researchers round the Earth. In the beginning, Nunziato and Cowin [23] developed a nonlinear theory of flexible material with voids. A few years later, after the conception of this hypothesis, the one-dimensional theory of flexible materials with voids has also been introduced by Cowin and Nunziato [24]. Chandrasekharaiah [25] addressed the characteristics of Rayleigh–Lamb waves in an elastic plate with voids. Likewise, the subject area of wave propagation in a micropolar elastic plate with voids has been investigated by Tomar [26]. Further, Iesan [27] gave a brief mathematical discussion of a theory of thermoviscoelastic materials with voids, whereas a problem on plane waves in a rotating generalized thermoelastic solid with voids has been discussed by Singh and Tomar [28]. Recently, Vishwakarma and Gupta [29] performed a case-wise study on the Rayleigh wave propagation under the effect of a rigid boundary. In their study, two cases have been discussed, out of which in case II the lower medium has been counted as a half-space with void pores.

This research study is fundamentally concerned with the real impact of thickness and initial stress on the propagation of Rayleigh waves through an anisotropic crustal layer lying over a half-space with void pores.

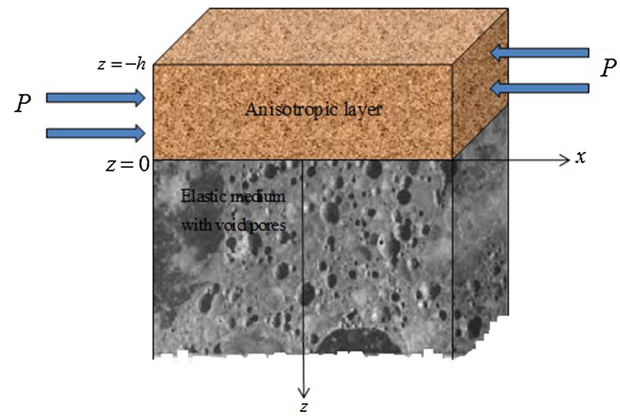

Fig. 1 Structure of the model

The displacement components have been deduced separately for both layer and half-space using governing equations of motion. With the assistance of the displacement components and suitable boundary conditions, the dispersion relation has been derived mathematically. The considerable influence of thickness and initial stress has been illustrated graphically not only for phase velocity, but also for attenuation of Rayleigh wave propagation by providing numerical values for different parameters.

2 Mathematical formulation of the problem

We have considered an anisotropic elastic layer of finite thickness h under horizontal initial stress P overlying a semi-infinite elastic half-space with void pores. We have also assumed that the x-axis is in the direction of wave propagation, i.e. along the horizontal direction with velocity c, and the z-axis is pointed vertically downwards. The free surface and interface between these two media are located at $z = -h$ and $z = 0$, respectively. ρ_1 is the density of the layer, whereas in case of half-space, rigidity and density are μ_2 and ρ_2, respectively. The complete geometrical layout of the problem is shown in Fig. 1. For Rayleigh waves, displacement is independent of y, and if (u, v, w) is the displacement at any point $P_1(x, y, z)$ in the medium, then $v = 0$ and u, w are functions of x, z, and t.

3 Fundamental equations and solution

3.1 Dynamics of the layer

For the propagation of Rayleigh waves in an anisotropic elastic layer under initial stress P, the governing equations given by Biot [30] are

$$\frac{\partial \tau_{xx}^{(1)}}{\partial x} + \frac{\partial \tau_{xz}^{(1)}}{\partial z} - P\frac{\partial \omega_{xz}}{\partial z} = \rho_1 \frac{\partial^2 u_1}{\partial t^2} \tag{3.1}$$

and

$$\frac{\partial \tau_{xz}^{(1)}}{\partial x} + \frac{\partial \tau_{zz}^{(1)}}{\partial z} - P\frac{\partial \omega_{xz}}{\partial x} = \rho_1 \frac{\partial^2 w_1}{\partial t^2} \tag{3.2}$$

where $\tau_{xx}^{(1)}$, $\tau_{zz}^{(1)}$, and $\tau_{xz}^{(1)}$ are the shearing stress components, $u_1(x, z, t)$ and $w_1(x, z, t)$ are the factors of displacement in the layer along x and z directions, respectively, and ω_{xz} is the rotational component defined by

$$\omega_{xz} = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} - \frac{\partial w_1}{\partial x}\right). \tag{3.3}$$

The stress–strain relations for the anisotropic layer given by Biot [30] are

$$\begin{aligned} \tau_{xx}^{(1)} &= \mu_{11}e_{xx} + \mu_{12}e_{yy} + \mu_{13}e_{zz} + 2\mu_{14}e_{yz} + 2\mu_{15}e_{xz} + 2\mu_{16}e_{xy}, \\ \tau_{zz}^{(1)} &= \mu_{31}e_{xx} + \mu_{32}e_{yy} + \mu_{33}e_{zz} + 2\mu_{34}e_{yz} + 2\mu_{35}e_{xz} + 2\mu_{36}e_{xy}, \end{aligned} \tag{3.4}$$

$$\text{and}\quad \tau_{xz}^{(1)} = \mu_{51}e_{xx} + \mu_{52}e_{yy} + \mu_{53}e_{zz} + 2\mu_{54}e_{yz} + 2\mu_{55}e_{xz} + 2\mu_{56}e_{xy}$$

where μ_{ij} ($i = 1, 3, 5$; $j = 1, 2, \ldots, 6$) are the elastic constants and e_{ij} are the incremental strain components defined by

$$e_{xx} = \frac{\partial u_1}{\partial x},\; e_{zz} = \frac{\partial w_1}{\partial z},\; e_{xz} = \frac{1}{2}\left(\frac{\partial u_1}{\partial z} + \frac{\partial w_1}{\partial x}\right) \quad \text{and} \quad e_{xy} = e_{yy} = e_{yz} = 0. \tag{3.5}$$

With the help of set of relations (3.3), (3.4), and (3.5), the equations of motion (3.1) and (3.2) expand to

$$\mu_{11}\frac{\partial^2 u_1}{\partial x^2} + \mu_{15}\frac{\partial^2 w_1}{\partial x^2} + \left(\mu_{55} - \frac{P}{2}\right)\frac{\partial^2 u_1}{\partial z^2} + \mu_{35}\frac{\partial^2 w_1}{\partial z^2} + 2\mu_{15}\frac{\partial^2 u_1}{\partial x \partial z} + \left(\mu_{13} + \mu_{55} + \frac{P}{2}\right)\frac{\partial^2 w_1}{\partial x \partial z} = \rho_1\frac{\partial^2 u_1}{\partial t^2}, \tag{3.6}$$

$$\text{and}\quad \mu_{15}\frac{\partial^2 u_1}{\partial x^2} + \left(\mu_{55} + \frac{P}{2}\right)\frac{\partial^2 w_1}{\partial x^2} + \mu_{35}\frac{\partial^2 u_1}{\partial z^2} + \mu_{33}\frac{\partial^2 w_1}{\partial z^2} + \left(\mu_{13} + \mu_{55} - \frac{P}{2}\right)\frac{\partial^2 u_1}{\partial x \partial z} + 2\mu_{35}\frac{\partial^2 w_1}{\partial x \partial z} = \rho_1\frac{\partial^2 w_1}{\partial t^2}. \tag{3.7}$$

Let the solutions to Eqs. (3.6) and (3.7) be $u_1(x, z, t) = F_1(z)e^{ik(x-ct)}$ and $w_1(x, z, t) = G_1(z)e^{ik(x-ct)}$, respectively. Using these values, in Eqs. (3.6) and (3.7), we obtain

$$\left[\left(\mu_{55} - \frac{P}{2}\right)D^2 + 2ik\mu_{15}D + \left(\rho_1 k^2 c^2 - \mu_{11}k^2\right)\right]F_1 + \left[\mu_{35}D^2 + ik\left(\mu_{13} + \mu_{55} + \frac{P}{2}\right)D - \mu_{15}k^2\right]G_1 = 0 \tag{3.8}$$

$$\text{and}\quad \left[\mu_{35}D^2 + ik\left(\mu_{13} + \mu_{55} - \frac{P}{2}\right)D - \mu_{15}k^2\right]F_1 + \left[\mu_{33}D^2 + 2ik\mu_{35}D + \left(\rho_1 k^2 c^2 - k^2\left(\mu_{55} + \frac{P}{2}\right)\right)\right]G_1 = 0 \tag{3.9}$$

where $D = \frac{\partial}{\partial z}$, $D^2 = \frac{\partial^2}{\partial z^2}$, k is the wave number, and c is the phase velocity.

Let us consider $F_1(z) = Me^{-ksz}$ and $G_1(z) = Ne^{-ksz}$, where M and N are arbitrary constants.

In order to resolve the above system of simultaneous linear equations with constant coefficients, we substitute $F_1(z)$ and $G_1(z)$ in Eqs. (3.8) and (3.9), and then the equations get reduced to

$$\left[\left(\mu_{55} - \frac{P}{2}\right)s^2 - 2i\mu_{15}s + \left(\rho_1 c^2 - \mu_{11}\right)\right]M + \left[\mu_{35}s^2 - i\left(\mu_{13} + \mu_{55} + \frac{P}{2}\right)s - \mu_{15}\right]N = 0 \tag{3.10}$$

$$\text{and}\quad \left[\mu_{35}s^2 - i\left(\mu_{13} + \mu_{55} - \frac{P}{2}\right)s - \mu_{15}\right]M + \left[\mu_{33}s^2 - 2i\mu_{35}s + \left(\rho_1 c^2 - \left(\mu_{55} + \frac{P}{2}\right)\right)\right]N = 0. \tag{3.11}$$

For getting the nontrivial solution of Eqs. (3.10) and (3.11), we may write

$$\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix} = 0 \tag{3.12}$$

where a_{11}, a_{12}, a_{21}, and a_{22} are defined in "Appendix I".

Solving the determinant given in (3.12), we get a polynomial equation in s with degree 4 as

$$a_0 s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4 = 0 \tag{3.13}$$

where $a_0,\ a_1,\ a_2,\ a_3$, and a_4 are defined in "Appendix I".

Suppose s_i $(i = 1, \ldots, 4)$ are the roots of Eq. (3.13) and from Eq. (3.10) corresponding to $s = s_i$, the ratio of the displacement components F_{1i} and G_{1i} is given by

$$\frac{G_{1i}}{F_{1i}} = \frac{N_i}{M_i} = -\frac{\left[\left(\mu_{55} - \frac{P}{2}\right) s_i^2 - 2\mathrm{i}\mu_{15} s_i + \left(\rho_1 c^2 - \mu_{11}\right)\right]}{\left[\mu_{35} s_i^2 - \mathrm{i}\left(\mu_{13} + \mu_{55} + \frac{P}{2}\right) s_i - \mu_{15}\right]} = n_i \tag{3.14}$$

where n_i, M_i, and N_i are arbitrary constants.

Finally, the desired solutions of Eqs. (3.6) and (3.7) can be written as

$$u_1(x,z,t) = \left(M_1 e^{-ks_1 z} + M_2 e^{-ks_2 z} + M_3 e^{-ks_3 z} + M_4 e^{-ks_4 z}\right) e^{\mathrm{i}k(x-ct)} \tag{3.15}$$

$$\text{and} \quad w_1(x,z,t) = \left(n_1 M_1 e^{-ks_1 z} + n_2 M_2 e^{-ks_2 z} + n_3 M_3 e^{-ks_3 z} + n_4 M_4 e^{-ks_4 z}\right) e^{\mathrm{i}k(x-ct)}. \tag{3.16}$$

3.2 Dynamics of the half-space

In the absence of body forces, the governing equations of motion for the homogeneous elastic half-space with void pores given by Cowin and Nunziato [24] are

$$\mu_2\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 u_2}{\partial z^2}\right) + (\lambda_2 + \mu_2)\left(\frac{\partial^2 u_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial x \partial z}\right) + \beta\frac{\partial \phi}{\partial x} = \rho_2 \frac{\partial^2 u_2}{\partial t^2}, \tag{3.17}$$

$$\mu_2\left(\frac{\partial^2 w_2}{\partial x^2} + \frac{\partial^2 w_2}{\partial z^2}\right) + (\lambda_2 + \mu_2)\left(\frac{\partial^2 w_2}{\partial z^2} + \frac{\partial^2 u_2}{\partial x \partial z}\right) + \beta\frac{\partial \phi}{\partial z} = \rho_2 \frac{\partial^2 w_2}{\partial t^2}, \tag{3.18}$$

$$\text{and} \quad \bar{\alpha}\left(\frac{\partial^2 \phi}{\partial x^2} + \frac{\partial^2 \phi}{\partial z^2}\right) - \bar{\omega}\frac{\partial \phi}{\partial t} - \xi\phi - \beta\left(\frac{\partial u_2}{\partial x} + \frac{\partial w_2}{\partial z}\right) = \rho_2 \bar{k}\frac{\partial^2 \phi}{\partial t^2} \tag{3.19}$$

where $u_2(x,z,t)$ and $w_2(x,z,t)$ are components of displacement along x and z directions, respectively. λ_2 is Lame's constant. The change between volume fraction and reference volume fraction is denoted by ϕ. The void parameters are $\bar{\alpha}$, $\bar{\omega}$, ξ, β, and $\bar{k}$.

For the elastic half-space with void pores, the stress–displacement relations given by Weiskopf [31] are

$$\tau_{xz}^{(2)} = \mu_2\left(\frac{\partial w_2}{\partial x} + \frac{\partial u_2}{\partial z}\right) \quad \text{and} \quad \tau_{zz}^{(2)} = \lambda_2 \frac{\partial u_2}{\partial x} + (\lambda_2 + 2\mu_2)\frac{\partial w_2}{\partial z} \tag{3.20}$$

where $\tau_{xz}^{(2)}$ and $\tau_{zz}^{(2)}$ are the shearing stress components.

Now, let us consider

$$u_2 = \frac{\partial L}{\partial x} - \frac{\partial Q}{\partial z} \quad \text{and} \quad w_2 = \frac{\partial L}{\partial z} + \frac{\partial Q}{\partial x}. \tag{3.21}$$

Using (3.21), Eqs. (3.17) and (3.18) get reduced to

$$\frac{\partial^2 L}{\partial x^2} + \frac{\partial^2 L}{\partial z^2} - \frac{1}{{\alpha_1}^2}\frac{\partial^2 L}{\partial t^2} = \bar{\beta}_1 \phi \tag{3.22}$$

$$\text{and} \quad \frac{\partial^2 Q}{\partial x^2} + \frac{\partial^2 Q}{\partial z^2} - \frac{1}{{\alpha_2}^2}\frac{\partial^2 Q}{\partial t^2} = 0 \tag{3.23}$$

where α_1, $\bar{\beta}_1$, and α_2 are defined in "Appendix I".

For the propagation of the wave in the positive direction of the x-axis with velocity c, the solutions of Eqs. (3.19), (3.22), and (3.23) are

$$\phi(x,z,t) = \bar{\phi}(z)e^{\mathrm{i}k(x-ct)}, \tag{3.24}$$
$$L(x,z,t) = \phi_0(z)e^{\mathrm{i}k(x-ct)}, \tag{3.25}$$
$$\text{and} \quad Q(x,z,t) = \psi_0(z)e^{\mathrm{i}k(x-ct)}, \text{ respectively.} \tag{3.26}$$

Substituting Eqs. (3.24) and (3.25) in Eqs. (3.19) and (3.22), we get

$$\bar{\alpha}\frac{d^2\bar{\phi}}{dz^2} + \bar{\phi}(\mathrm{i}\bar{\omega}kc - k^2\bar{\alpha} - \xi + \rho_2\bar{k}k^2c^2) - \beta\frac{d^2\phi_0}{dz^2} + \beta k^2\phi_0 = 0 \tag{3.27}$$
$$\text{and} \qquad \frac{d^2\phi_0}{dz^2} - k^2\bar{a}^2\phi_0 = \bar{\beta}_1\bar{\phi} \tag{3.28}$$

where $\bar{a}$ is defined in “Appendix I”.

In direction of starting out the results of Eqs. (3.27) and (3.28), let us assume

$$\phi_0(z) = Ae^{-pz} + Be^{pz}$$
$$\text{and} \quad \bar{\phi}(z) = Ce^{-pz} + De^{pz}$$

where A, B, C, and D are arbitrary constants.

Replacing the values of $\phi_0(z)$ and $\bar{\phi}(z)$, Eqs. (3.27) and (3.28) get transformed into the set of simultaneous equations

$$(p^2 - k^2\bar{a}^2)A - \bar{\beta}_1 C = 0, \tag{3.29}$$
$$(p^2 - k^2\bar{a}^2)B - \bar{\beta}_1 D = 0, \tag{3.30}$$
$$(\beta k^2 - \beta p^2)A + (\bar{\alpha}p^2 + \mathrm{i}\bar{\omega}kc - k^2\bar{\alpha} - \xi + \rho_1\bar{k}k^2c^2)C = 0, \tag{3.31}$$
$$\text{and} \quad (\beta k^2 - \beta p^2)B + (\bar{\alpha}p^2 + \mathrm{i}\bar{\omega}kc - k^2\bar{\alpha} - \xi + \rho_1\bar{k}k^2c^2)D = 0. \tag{3.32}$$

For getting a nontrivial solution of Eqs. (3.29) , (3.30) , (3.31), and (3.32), we may write

$$\begin{vmatrix} b_{11} & 0 & b_{13} & 0 \\ 0 & b_{11} & 0 & b_{13} \\ b_{31} & 0 & b_{33} & 0 \\ 0 & b_{31} & 0 & b_{33} \end{vmatrix} = 0 \tag{3.33}$$

where b_{11}, b_{13}, b_{31}, and b_{33} are defined in “Appendix I”.

On expanding and solving the determinant given in (3.33), we get a polynomial equation in p with degree 4, with four distinct roots, namely

$$p_1 = \pm\sqrt{\xi_1 - \frac{\sqrt{\eta_1 - \gamma_1}}{q_1}} \tag{3.34}$$
$$\text{and} \quad p_2 = \pm\sqrt{\xi_1 + \frac{\sqrt{\eta_1 - \gamma_1}}{q_1}} \tag{3.35}$$

where ξ_1, η_1, γ_1, and q_1 are well defined in “Appendix I”.

As we are concerned with the traversal of the wave in half-space, so as $z \to \infty$, we have $\phi_0(z) \to 0$ and $\bar{\phi}(z) \to 0$. Due to this reason, neglecting positive roots of (3.33), we get

$$\phi_0(z) = A_1e^{-p_1z} + A_2e^{-p_2z} \tag{3.36}$$
$$\text{and} \quad \bar{\phi}(z) = C_1e^{-p_1z} + C_2e^{-p_2z} = m_1A_1e^{-p_1z} + m_2A_2e^{-p_2z} \tag{3.37}$$

where for $i = 1, 2$, $\frac{C_i}{A_i} = \frac{{p_i}^2 - k^2\bar{a}^2}{\bar{\beta}_1} = m_i$, and m_i, A_i, and C_i are arbitrary constants.

Putting the value of Q from (3.26) into Eq. (3.23), we have

$$\frac{d^2\psi_0}{dz^2} - k^2\bar{b}^2\psi_0(z) = 0 \tag{3.38}$$

where $\bar{b}$ is defined in "Appendix I".

We know that as $z \to \infty$ we have $\psi_0(z) \to 0$, so the solution of Eq. (3.38) is

$$\psi_0(z) = A_3 e^{-\bar{b}kz} \tag{3.39}$$

where A_3 is an arbitrary constant.

Now, substituting the values of $\bar{\phi}(z)$, $\phi_0(z)$, and $\psi_0(z)$ in relations (3.24), (3.25), and (3.26), we get

$$\phi(x, z, t) = (m_1 A_1 e^{-p_1 z} + m_2 A_2 e^{-p_2 z})e^{\mathrm{i}k(x-ct)}, \tag{3.40}$$

$$L(x, z, t) = (A_1 e^{-p_1 z} + A_2 e^{-p_2 z})e^{\mathrm{i}k(x-ct)}, \tag{3.41}$$

$$\text{and} \quad Q(x, z, t) = A_3 e^{-\bar{b}kz} e^{\mathrm{i}k(x-ct)}. \tag{3.42}$$

Substituting the values of L and Q in (3.21), we get the displacement components as

$$u_2(x, z, t) = \left[\mathrm{i}k(A_1 e^{-p_1 z} + A_2 e^{-p_2 z}) + \bar{b}kA_3 e^{-\bar{b}kz}\right]e^{\mathrm{i}k(x-ct)} \tag{3.43}$$

$$\text{and} \quad w_2(x, z, t) = \left[-(A_1 p_1 e^{-p_1 z} + A_2 p_2 e^{-p_2 z}) + \mathrm{i}kA_3 e^{-\bar{b}kz}\right]e^{\mathrm{i}k(x-ct)}. \tag{3.44}$$

4 Boundary conditions and dispersion relation

Continuity of displacement and shearing stress components at the interface of the layer and half-space, the stress-free case at the free surface of the layer and the presence of void pores provide suitable boundary conditions as:

(i) At the interface $z = 0$, the displacement components are continuous,

$$i.e. \; u_1 = u_2 \quad \text{and} \; w_1 = w_2.$$

(ii) Again at the interface $z = 0$, the shearing components of stresses are continuous,

$$i.e. \; \tau_{xz}^{(1)} = \tau_{xz}^{(2)} \quad \text{and} \; \tau_{zz}^{(1)} = \tau_{zz}^{(2)}.$$

(iii) At the upper boundary plane (free surface) $z = -h$, the shearing stress components vanish,

$$i.e. \; \tau_{xz}^{(1)} = 0 \quad \text{and} \; \tau_{zz}^{(1)} = 0.$$

(iv) As the lower medium is considered as homogeneous elastic half-space with void pores, so at $z = 0$, we can write

$$\mathbf{n} \cdot \nabla\phi = 0.$$

Now using the four boundary conditions and Eqs. (3.15), (3.16), (3.40), (3.43), and (3.44) simultaneously, we reach the set of equations

$$M_1 + M_2 + M_3 + M_4 - \mathrm{i}kA_1 - \mathrm{i}kA_2 - \bar{b}kA_3 = 0, \tag{4.1}$$

$$n_1 M_1 + n_2 M_2 + n_3 M_3 + n_4 M_4 + p_1 A_1 + p_2 A_2 - \mathrm{i}kA_3 = 0, \tag{4.2}$$

$$a_{31} M_1 + a_{32} M_2 + a_{33} M_3 + a_{34} M_4 + a_{35} A_1 + a_{36} A_2 + a_{37} A_3 = 0, \tag{4.3}$$

$$a_{41}M_1 + a_{42}M_2 + a_{43}M_3 + a_{44}M_4 + a_{45}A_1 + a_{46}A_2 + a_{47}A_3 = 0, \tag{4.4}$$

$$a_{51}M_1 + a_{52}M_2 + a_{53}M_3 + a_{54}M_4 = 0, \tag{4.5}$$

$$a_{61}M_1 + a_{62}M_2 + a_{63}M_3 + a_{64}M_4 = 0, \tag{4.6}$$

$$\text{and} \;\; p_1 m_1 A_1 + p_2 m_2 A_2 = 0 \tag{4.7}$$

where the coefficients a_{31} to a_{37}, a_{41} to a_{47}, a_{51} to a_{54}, and a_{61} to a_{64} are well defined in "Appendix I".

Eliminating M_1, M_2, M_3, M_4, A_1, A_2, and A_3 from Eqs. (4.1) to (4.7), we get

$$\Delta(k, c) = \begin{vmatrix} 1 & 1 & 1 & 1 & -\mathrm{i}k & -\mathrm{i}k & -\bar{b}k \\ n_1 & n_2 & n_3 & n_4 & p_1 & p_2 & -\mathrm{i}k \\ a_{31} & a_{32} & a_{33} & a_{34} & a_{35} & a_{36} & a_{37} \\ a_{41} & a_{42} & a_{43} & a_{44} & a_{45} & a_{46} & a_{47} \\ a_{51} & a_{52} & a_{53} & a_{54} & 0 & 0 & 0 \\ a_{61} & a_{62} & a_{63} & a_{64} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_1 m_1 & p_2 m_2 & 0 \end{vmatrix} = 0. \tag{4.8}$$

Equation (4.8) is the desired dispersion equation for Rayleigh wave propagation in an anisotropic elastic layer under initial stress overlying an elastic half-space with void pores.

5 Particular cases

Case I

Considering $\mu_{15} = \mu_{35} = 0$, then Eq. (4.8) gets converted to

$$\begin{vmatrix} 1 & 1 & 1 & 1 & -\mathrm{i}k & -\mathrm{i}k & -\bar{b}k \\ n_{11} & n_{22} & n_{33} & n_{44} & p_1 & p_2 & -\mathrm{i}k \\ a_{311} & a_{322} & a_{333} & a_{344} & a_{35} & a_{36} & a_{37} \\ a_{411} & a_{422} & a_{433} & a_{444} & a_{45} & a_{46} & a_{47} \\ a_{511} & a_{522} & a_{533} & a_{544} & 0 & 0 & 0 \\ a_{611} & a_{622} & a_{633} & a_{644} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_1 m_1 & p_2 m_2 & 0 \end{vmatrix} = 0 \tag{5.1}$$

where the coefficients n_{ii}, a_{3ii}, a_{4ii}, a_{5ii}, and a_{6ii} (for $i = 1, 2, 3, 4$) are defined in "Appendix II".

Equation (5.1) is the dispersion relation for Rayleigh wave propagation in a pre-stressed orthotropic layer resting over a half-space with void pores.

Case II

Considering $\mu_{11} = \mu_{33} = \lambda_1 + 2\mu_1$, $\mu_{13} = \lambda_1$, $\mu_{55} = \mu_1$ and $\mu_{15} = \mu_{35} = 0$, then Eq. (4.8) gets converted to

$$\begin{vmatrix} 1 & 1 & 1 & 1 & -\mathrm{i}k & -\mathrm{i}k & -\bar{b}k \\ n_{111} & n_{222} & n_{333} & n_{444} & p_1 & p_2 & -\mathrm{i}k \\ a_{3111} & a_{3222} & a_{3333} & a_{3444} & a_{35} & a_{36} & a_{37} \\ a_{4111} & a_{4222} & a_{4333} & a_{4444} & a_{45} & a_{46} & a_{47} \\ a_{5111} & a_{5222} & a_{5333} & a_{5444} & 0 & 0 & 0 \\ a_{6111} & a_{6222} & a_{6333} & a_{6444} & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & p_1 m_1 & p_2 m_2 & 0 \end{vmatrix} = 0 \tag{5.2}$$

where the coefficients n_{iii}, a_{3iii}, a_{4iii}, a_{5iii}, and a_{6iii} (for $i = 1, 2, 3, 4$) are defined in "Appendix II".

Equation (5.2) is the dispersion relation for Rayleigh wave propagation in a pre-stressed isotropic layer resting over a half-space with void pores.

Case III

Taking $h = 0$, then Eq. (4.8) gets reduced to

$$\begin{vmatrix} a_{35} & a_{36} & a_{37} \\ a_{45} & a_{46} & a_{47} \\ p_1 m_1 & p_2 m_2 & 0 \end{vmatrix} = 0. \tag{5.3}$$

Equation (5.3) is the dispersion relation for Rayleigh waves traversing in a half-space with void pores.

Case IV

In the absence of initial stress, i.e. $P = 0$, and void parameters, i.e. $\bar{\alpha} = \bar{\omega} = \xi = \beta = \bar{k} = 0$, Eq. (4.8) gets reduced to

$$\begin{vmatrix} 1 & 1 & 1 & 1 & -1 & -1 \\ n_1^{'} & n_2^{'} & n_3^{'} & n_4^{'} & -m_1^{'} & -m_2^{'} \\ a_{31}^{'} & a_{32}^{'} & a_{33}^{'} & a_{34}^{'} & a_{35}^{'} & a_{36}^{'} \\ a_{41}^{'} & a_{42}^{'} & a_{43}^{'} & a_{44}^{'} & a_{45}^{'} & a_{46}^{'} \\ a_{51}^{'} & a_{52}^{'} & a_{53}^{'} & a_{54}^{'} & 0 & 0 \\ a_{61}^{'} & a_{62}^{'} & a_{63}^{'} & a_{64}^{'} & 0 & 0 \end{vmatrix} = 0 \tag{5.4}$$

where $n_i^{'}$ (for $i = 1, \ldots, 4$), $m_1^{'}, m_2^{'}, a_{3i}^{'}$ (for $i = 1, \ldots, 6$), $a_{4i}^{'}$ (for $i = 1, \ldots, 6$), $a_{5i}^{'}$ (for $i = 1, \ldots, 4$) and $a_{6i}^{'}$ (for $i = 1, \ldots, 4$) are defined in "Appendix III".

Equation (5.4) is the dispersion relation of Rayleigh waves propagating through an anisotropic layer resting over an elastic half-space. This equation is in well agreement with the dispersion relation established by Pal et al. [15], considering $\eta = 1$ in equation (34) of their study, i.e. when the half-space is considered to be an elastic half-space instead of a sandy one.

Case V

In the absence of the superficial layer, i.e. $h = 0$, and void parameters, i.e. $\bar{\alpha} = \bar{\omega} = \xi = \beta = \bar{k} = 0$, Eq. (4.8) gets converted to

$$\begin{vmatrix} a_{355}^{'} & a_{366}^{'} \\ a_{455}^{'} & a_{466}^{'} \end{vmatrix} = 0 \tag{5.5}$$

where $a_{355}^{'}, a_{366}^{'}, a_{455}^{'}$ and $a_{466}^{'}$ are defined in "Appendix III".

Equation (5.5) is the dispersion relation of Rayleigh waves traversing through an elastic half-space. This relation coincides with the dispersion relation obtained by Pal et al. [15] in the special case IV of their study.

6 Numerical results and discussion

On the basis of dispersion relation (4.8), an extensive survey has been held out to shed light on the propagation characteristics of Rayleigh waves in an initially stressed anisotropic layer lying over a half-space with void pores. Generally, the wave number k is considered to be complex and thus can be manifested as $k = k_1 + \mathrm{i}k_2 = k_1(1 + \mathrm{i}\delta)$, where k_1 and k_2 are real entities and $\delta = \frac{\mathrm{Im}[k]}{\mathrm{Re}[k]} = \frac{k_2}{k_1} (\ll 1)$ is termed as the attenuation coefficient. Hence, the phase velocity of Rayleigh waves c can be expressed as $c = \frac{\omega}{k_1}$. The dispersion Eq. (4.8) is separated into real and imaginary components, and thus contributing the following relations:

$$\mathrm{Re}[\Delta(k_1, c, \delta)] = 0 \tag{6.1}$$

$$\text{and } \mathrm{Im}[\Delta(k_1, c, \delta)] = 0. \tag{6.2}$$

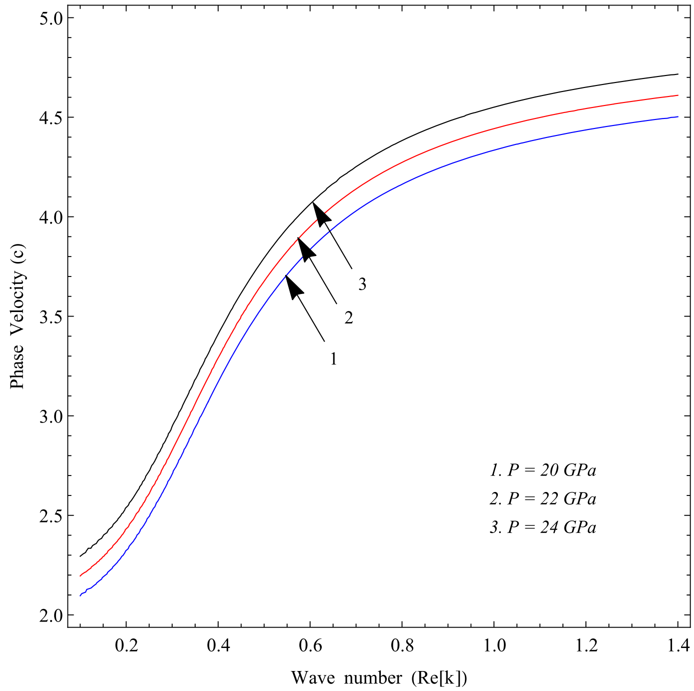

Fig. 2 Effect of initial stress P on the phase velocity of Rayleigh waves. Here, $\delta = 0.001$

Equations (6.1) and (6.2) generate the dispersion or phase velocity curves (i.e. c versus Re[k]) and the attenuation curves (i.e. Log(δ) versus Re[k]), respectively. In usual practice, solving Eqs. (6.1) and (6.2) by an analytical method is infeasible. Further, finding a solution of these equations numerically is also very strenuous. Hence, in order to find numerical values of c and δ from Eqs. (6.1) and (6.2), the following steps of an iterative technique given by Ke et al. [32] have been applied:

(i) Equation (6.1) has been solved for c by taking $\delta = 0$ and fixing k_1 or Re[k] at a particular value.
(ii) Equation (6.2) has been solved for getting δ by using the numerical value of c obtained from step (i).
(iii) Equation (6.1) has been again solved for c with the help of δ obtained from step (ii).
(iv) Steps (ii) and (iii) have been reiterated till the numerical values of c and δ obtained from two successive iterations are within the expected span of errors.

The values of c and δ obtained by following the aforementioned steps have been utilized to plot the phase velocity and attenuation curves, respectively. Numerical computations and plotting of Figures have been carried out with the aid of the software Wolfram Mathematica (Version 9.0).

The most influencing parameters encountered in the presumed geometry are initial stress P and thickness h of the stratum. The strong influence of these parameters on phase velocity and attenuation coefficient of Rayleigh waves is depicted in Figs. 2, 3, 4, and 5. Moreover, to understand the effect of initial stress and thickness on phase velocity and attenuation curves, a comparative study has been carried out considering anisotropic, orthotropic, and isotropic strata, which is irradiated by Figs. 6, 7, 8, and 9. Also, Fig. 10 is plotted to validate our obtained numerical results with a pre-existing special case in absence of initial stress. For computational purposes, the following data have been considered:

(a) For a pre-stressed anisotropic layer (Rasolofosaon and Zinszner [33]):

$\mu_{11} = 106.8$ GPa, $\mu_{12} = 27.1$ GPa, $\mu_{13} = 9.68$ GPa, $\mu_{14} = -0.03$ GPa, $\mu_{15} = 0.28$ GPa,
$\mu_{16} = 0.12$ GPa, $\mu_{32} = 18.22$ GPa, $\mu_{33} = 54.57$ GPa, $\mu_{34} = 2.44$ GPa, $\mu_{35} = -1.69$ GPa,
$\mu_{36} = -0.75$ GPa, $\mu_{52} = 0.13$ GPa, $\mu_{54} = 1.98$ GPa, $\mu_{55} = 25.03$ GPa, $\mu_{56} = 1.44$ GPa,
$\rho_1 = 2727$ Kg/m^3;

(b) For a half-space with void pores (Puri and Cowin [34]):

$\bar{\omega} = 0.008$ GPa, $\beta = 10$ GPa, $\xi = 12$ GPa, $\bar{\alpha} = 8$ GPa, $\bar{k} = 5$ GPa, $\lambda_2 = 15$ GPa, $\mu_2 = 7.5$ GPa, $\rho_2 = 2778$ Kg/m^3.

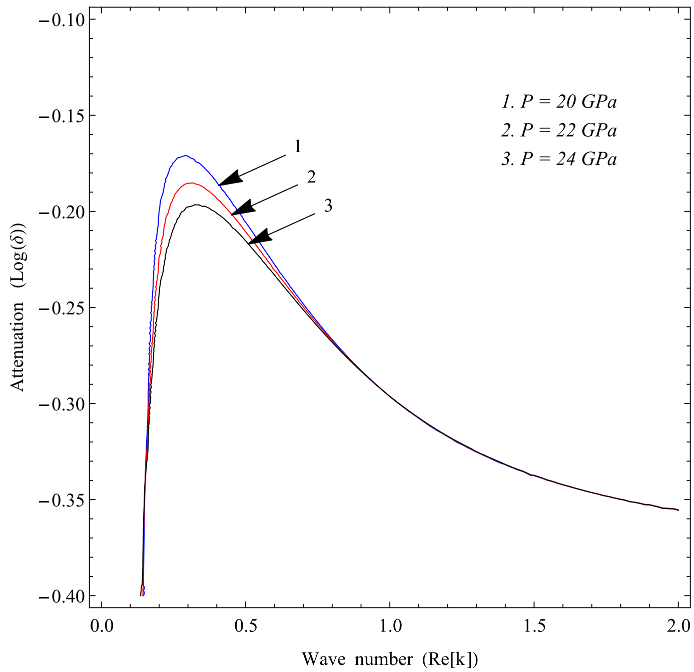

Fig. 3 Effect of initial stress P on the attenuation coefficient of Rayleigh waves. Here, $c = 2.5$ Km/s and $h = 0.1$ Km

6.1 Influence of initial stress

Figures 2 and 3 exemplify the impact of initial stress associated with the layer on the dispersion curves (c vs. Re[k]) and attenuation curves (Log(δ) vs. Re[k]), respectively. The numerical values assigned to the initial stress P are 20 GPa, 22 GPa, and 24 GPa for curves 1, 2, and 3, simultaneously. In the presumed geometry considering an anisotropic layer overlying a half-space with void pores, as the wave number (Re[k]) proceeds towards higher values, the phase velocity of Rayleigh waves increases. But, with the growing value of wave number, the attenuation exhibits a rising trend, attains a peak value, and then finally diminishes.

Moreover, it is remarked from Fig. 2 that the influence of initial stress on the dispersion curves is substantial for the considered range of the wave number. The dispersion curves shift upwards as the magnitude of initial stress rises, clearly depicting the fact that the traversal of Rayleigh waves becomes faster with the increment in initial stress present in the layer.

Contrary to this, Fig. 3 elucidates the fact that the initial stress does not possess a prominent effect on the attenuation of Rayleigh waves in the entire frequency regime of wave number. In the neighbourhood of peak values of the attenuation curves, a rising value of initial stress disfavours the growth of the attenuation coefficient. Further, for higher values of wave number and increasing magnitude of initial stress, the attenuation of Rayleigh waves becomes constant.

6.2 Influence of thickness of the layer

Figures 4 and 5 disclose the influence of thickness (h) of the layer on the dispersion and attenuation curves, respectively. The curves 1, 2, and 3 are drawn for $h = 0.1$ Km, $h = 0.2$ Km, and $h = 0.3$ Km, respectively.

It is noted from Fig. 4 that with the increment in thickness of the layer the propagation of Rayleigh waves becomes faster. In accession to the rising magnitude of thickness, if the initial stress (i.e. $P = 20$ GPa) is also exerted on the layer, then the curves 1, 2, and 3 shift upwards, and the propagation velocity gets more enhanced as compared to the stress-free case (i.e. $P = 0$).

Moreover, Fig. 5 irradiates the fact that in the absence of initial stress (i.e. $P = 0$), the attenuation curves 1, 2, and 3 (broken curves) indicate a decreasing trend with growing values of both wave number and thickness of the layer. Contrary to this, when the initial stress (i.e. $P = 20$ GPa) has been exerted horizontally on the layer, the attenuation curves exhibit different behaviours. All the curves 1, 2, and 3 rise up suddenly, attain

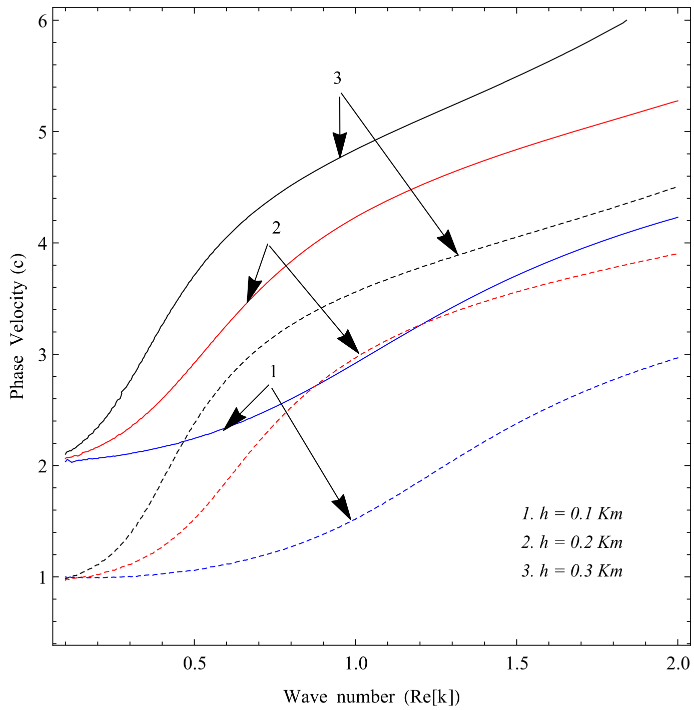

Fig. 4 Plots of the Rayleigh wave velocity that is illustrated by the real part of the dispersion Eq. (4.8) for different values of thickness h of the layer. The full curve represents the case when the initial stress is taken into account ($P = 20$ GPa), and the broken curve represents the absence of initial stress in the layer (i.e. $P = 0$). Here, $\delta = 0.001$

a maximum value and then diminish, as the wave number increases. The common fact that has been noticed in both the cases is that the attenuation curves tend to get closer to each other with increasing the value of thickness. Further, in the presence of initial stress, the attenuation coefficient is always lower than in the stress-free case.

6.3 Comparative study

Two different layered Earth models have been considered for comparison with the present study. These are a pre-stressed orthotropic layer overlying a half-space with void pores and pre-stressed isotropic layer overlying a half-space with void pores. The dispersion relations for the above-mentioned geometries are obtained in particular cases I and II, respectively. A comparative study has been carried out by plotting different curves for anisotropic, orthotropic, and isotropic strata which is depicted in Figs. 6, 7, 8, and 9.

In Figs. 6 and 7, the influence of initial stress has been shown on the phase velocity and attenuation of Rayleigh waves traversing through anisotropic, orthotropic, and isotropic layers. It is observed from both the Figures that an increasing value of initial stress exhibits opposite trends on phase velocity and attenuation coefficient. When the layer is anisotropic, with the increment in initial stress, the phase velocity increases, whereas when the layer is orthotropic or isotropic, the phase velocity decreases. For a particular amount of initial stress, the phase velocity is highest for an anisotropic layer, whereas lowest for an orthotropic layer.

This trend gets altered for the attenuation of Rayleigh waves, i.e. for an anisotropic layer, a growing value of initial stress disfavours the growth of attenuation, but for orthotropic or isotropic layers favours the growth of attenuation. For a fixed value of initial stress, attenuation is maximum for an orthotropic stratum, whereas minimum for an anisotropic one.

Considering anisotropic, orthotropic, and isotropic strata one by one, the impact of thickness on phase velocity and attenuation is exhibited in Figs. 8 and 9, respectively. It is noticed from Fig. 8 that as the thickness of the stratum increases, the phase velocity of Rayleigh waves traversing through orthotropic or isotropic strata diminishes, while this trend of phase velocity gets reversed for an anisotropic stratum, i.e. the phase velocity enhances with the increment in thickness of the stratum. Moreover, Fig. 9 elucidates the fact that a rising value of thickness possesses unfavourable influence on the attenuation coefficient of Rayleigh waves for anisotropic, orthotropic, and isotropic cases.

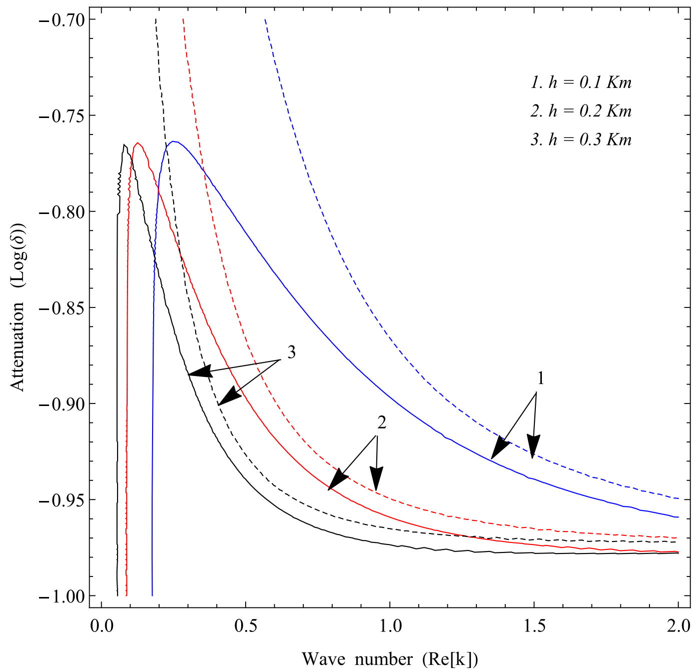

Fig. 5 Plots of the attenuation coefficient which is illustrated by the imaginary part of the dispersion Eq. (4.8) for different values of the thickness h of the layer. The full curve represents the case when initial stress is taken into account ($P = 20$ GPa), and the broken curve represents the absence of initial stress in the layer (i.e. $P = 0$). Here, $c = 2.5$ Km/s

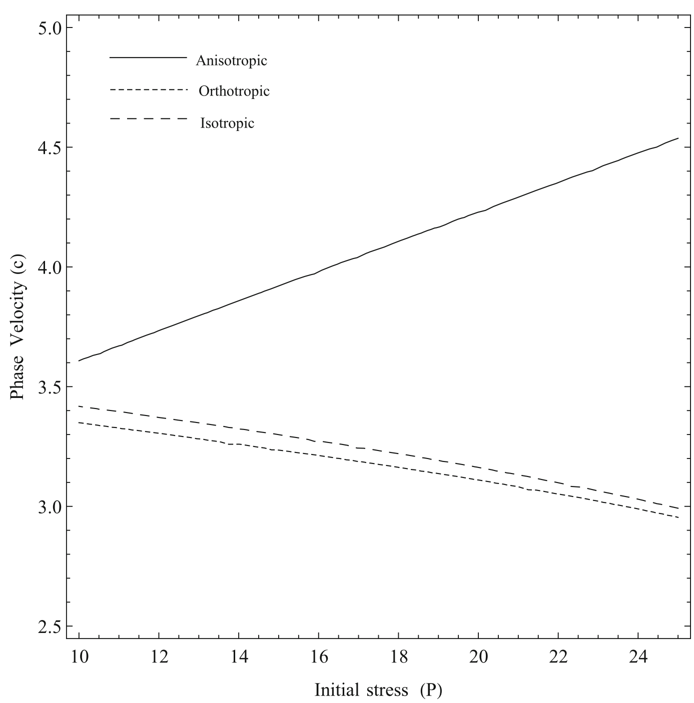

Fig. 6 Effect of initial stress P on the phase velocity of Rayleigh waves for anisotropic, orthotropic, and isotropic layers. Here, $\delta = 0.001$ and $h = 0.1$ Km

6.4 Validation of the numerical results

For the validation of the obtained results of our study with a pre-established special case, Fig. 10 is plotted depicting the pattern of phase velocity of Rayleigh waves. It is observed from Fig. 10 that in the absence of

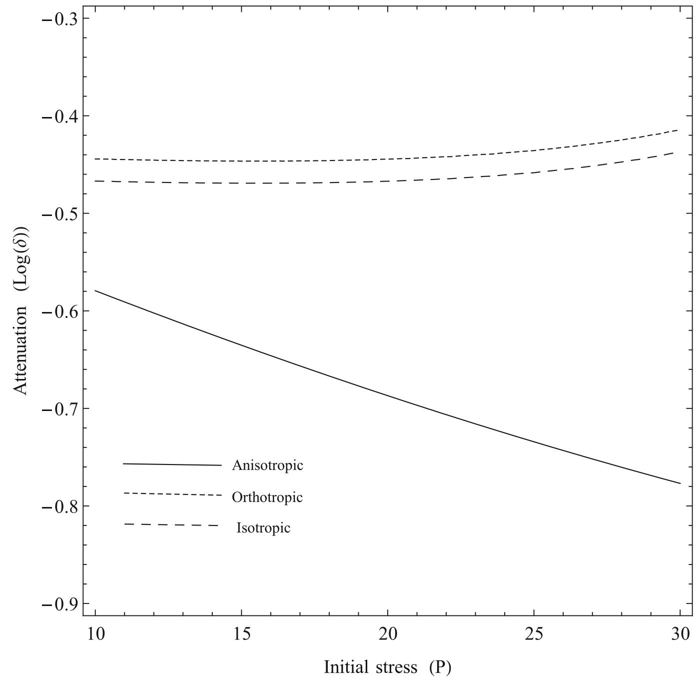

Fig. 7 Effect of initial stress P on the attenuation coefficient of Rayleigh waves for anisotropic, orthotropic, and isotropic layers. Here, $c = 2.5$ Km/s and $h = 0.1$ Km

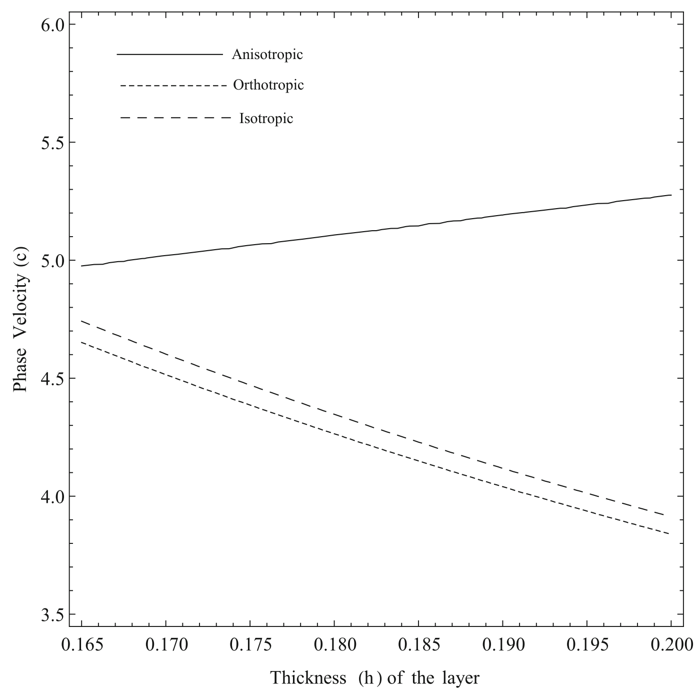

Fig. 8 Effect of the thickness h on phase velocity of Rayleigh waves for anisotropic, orthotropic, and isotropic layers. Here, $\delta = 0.001$ and $P = 20$ GPa

initial stress and void parameters the phase velocity of Rayleigh waves increases. This result obtained from Fig. 10 coincides with the pre-existing result (i.e. increasing trend exhibited by the phase velocity curve 1 for $\eta = 1$ in Fig. 3) obtained by Pal et al. [15]. Hence, it is clear from both the scenarios that the propagation of Rayleigh waves becomes faster through an anisotropic layer lying over an isotropic half-space.

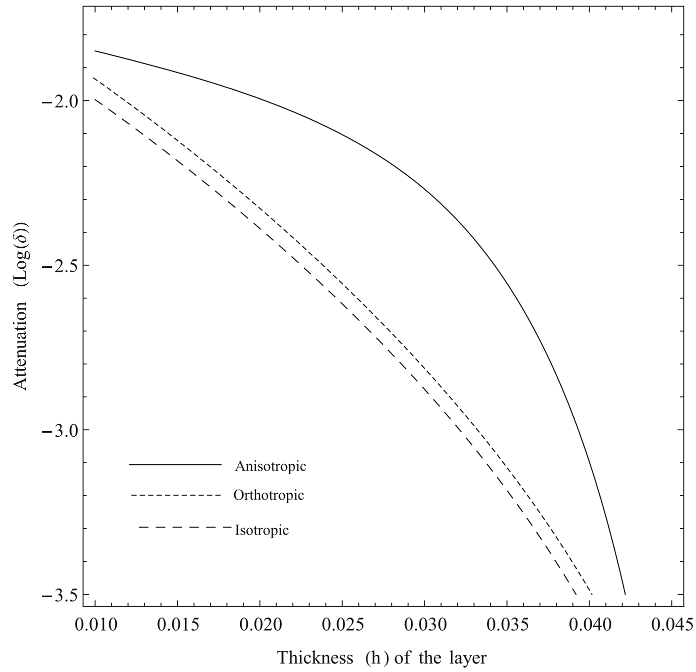

Fig. 9 Effect of thickness h on the attenuation coefficient of Rayleigh waves for anisotropic, orthotropic, and isotropic layers. Here, $c = 2.5$ Km/s and $P = 20$ GPa

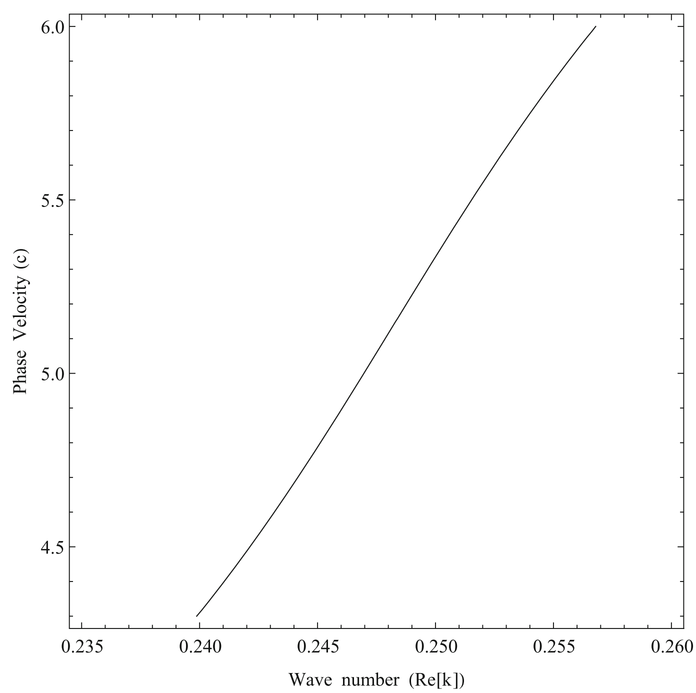

Fig. 10 Variation of the phase velocity with respect to the wave number for an anisotropic layer, in the absence of initial stress, i.e. $P = 0$ and void parameters, i.e. $\bar{\alpha} = \bar{\omega} = \xi = \beta = \bar{k} = 0$. Here, $\delta = 0.001$ and $h = 0.1$ Km

7 Concluding remarks

The main objective of the present work is to unfold the impact of initial stress and thickness on the propagation characteristics of Rayleigh waves in an anisotropic layer overlying a half-space with void pores. Mathematical expressions for the displacements in layer and half-space have been deduced separately. Further, using suitable boundary conditions along with the displacements in layer and half-space, the dispersion relation has been obtained in determinant form. With the aid of the dispersion relation, five particular cases have also been discussed. The phase velocity and attenuation of Rayleigh waves are found to be significantly influenced by initial stress and thickness of the layer which have been well exhibited graphically. Apart from the traversal of Rayleigh waves in an anisotropic layer, a graphical analysis has also been accomplished for studying the propagation behaviour of these waves in orthotropic and isotropic layers, separately. The most important highlights of the present study are summarized below:

- An increment in the magnitude of the wave number elevates the growth of the phase velocity, i.e. as the wave number proceeds towards higher values, the propagation of Rayleigh waves becomes faster. But this trend gets changed in case of attenuation, i.e. for increasing value of the wave number, initially the attenuation increases, attains a peak value, and then finally diminishes.
- When the layer is anisotropic, a rising value of initial stress exerted on the layer possesses favourable impact on phase velocity and unfavourable impact on the attenuation of Rayleigh waves, while the influence of initial stress gets altered when the considered layer is an orthotropic or isotropic one, i.e. with increasing value of initial stress, the phase velocity decreases whereas the attenuation increases.
- For a specific value of thickness or initial stress, the phase velocity of Rayleigh waves propagating through an anisotropic layer is highest, whereas lowest while propagating through an orthotropic layer.
- Attenuation always decreases with the increase in thickness, when Rayleigh waves traverse through anisotropic, orthotropic, and isotropic layers.
- Magmatic or igneous rocks account for more than 90% of the Earth's crust. These are highly anisotropic in nature. Some of these rocks exhibit vesicular texture (i.e. holes or pores), too. Due to the rapid cooling of magma, some of the dissolved gases are unable to emanate, and thus vesicles or pores are formed in these rocks. This work may serve as a helpful tool in the interpretation of data in seismic prospecting techniques used in the regions where igneous rocks are found.

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Appendix I

$$a_{11} = \left(\mu_{55} - \tfrac{P}{2}\right) s^2 - 2\mathrm{i}\mu_{15} s + \left(\rho_1 c^2 - \mu_{11}\right),$$

$$a_{12} = \mu_{35} s^2 - \mathrm{i}\left(\mu_{13} + \mu_{55} + \tfrac{P}{2}\right) s - \mu_{15},$$

$$a_{13} = \mu_{35} s^2 - \mathrm{i}\left(\mu_{13} + \mu_{55} - \tfrac{P}{2}\right) s - \mu_{15},$$

$$a_{14} = \mu_{33} s^2 - 2\mathrm{i}\mu_{35} s + \left(\rho_1 c^2 - (\mu_{55} + \tfrac{P}{2})\right),$$

$$a_0 = \tfrac{1}{2}\left(-2\mu_{35}^2 - \mu_{33}(P - 2\mu_{55})\right),$$

$$a_1 = \tfrac{1}{2}\left(-4\mathrm{i}\mu_{15}\mu_{33} + 2\mathrm{i}P\mu_{35} + 4\mathrm{i}\mu_{13}\mu_{35}\right),$$

$$a_2 = \tfrac{1}{2}\left(2\mu_{13}^2 - 2\mu_{11}\mu_{33} - 4\mu_{15}\mu_{35} + 4\mu_{13}\mu_{55} - c^2 P\rho_1 + 2c^2(\mu_{13} + \mu_{55})\rho_1\right),$$

$$a_3 = \tfrac{1}{2}\left(2\mathrm{i}P\mu_{15} - 4\mathrm{i}\mu_{13}\mu_{15} + 4\mathrm{i}\mu_{11}\mu_{35} - 4\mathrm{i}c^2\mu_{15}\rho_1 - 4\mathrm{i}c^2\mu_{35}\rho_1\right),$$

$$a_4 = \tfrac{1}{2}\left(P\mu_{11} - 2\mu_{15}^2 + 2\mu_{11}\mu_{55} - c^2 P\rho_1 - 2c^2\mu_{11}\rho_1 - 2c^2\mu_{55}\rho_1 + 2c^4\rho_1^2\right),$$

$$\alpha_1 = \sqrt{\tfrac{\lambda_2 + 2\mu_2}{\rho_2}},$$

$$\bar{\beta}_1 = -\frac{\beta}{\lambda_2 + 2\mu_2},$$

$$\alpha_2 = \sqrt{\frac{\mu_2}{\rho_2}},$$

$$\bar{a} = \sqrt{1 - \frac{c^2}{\alpha_1^2}},$$

$$b_{11} = p^2 - k^2\bar{a}^2,$$
$$b_{13} = -\bar{\beta}_1,$$
$$b_{31} = \beta k^2 - \beta p^2,$$
$$b_{33} = \bar{\alpha} p^2 + \mathrm{i}\bar{\omega} kc - k^2\bar{\alpha} - \xi + \rho_1 \bar{k} k^2 c^2,$$
$$\xi_1 = \frac{k^2}{2} + \frac{\bar{a}^2 k^2}{2} + \frac{\xi}{2\bar{\alpha}} - \frac{\mathrm{i}ck\bar{\omega}}{2\bar{\alpha}} + \frac{\beta\bar{\beta}_1}{2\bar{\alpha}} - \frac{c^2 k^2 \bar{k}\rho_2}{2\bar{\alpha}},$$
$$\eta_1 = \left(-k^2\bar{\alpha} - \bar{a}^2 k^2 \bar{\alpha} - \xi + \mathrm{i}ck\bar{\omega} - \beta\bar{\beta}_1 + c^2 k^2 \bar{k}\rho_2\right)^2,$$
$$\gamma_1 = 4\bar{\alpha}\left(\bar{a}^2 k^4 \bar{\alpha} + \bar{a}^2 k^2 \xi - \mathrm{i}\bar{a}^2 c k^3 \bar{\omega} + k^2 \beta\bar{\beta} - \bar{a}^2 c^2 k^4 \bar{k}\rho_2\right),$$
$$q_1 = 2\bar{\alpha},$$
$$\bar{b} = \sqrt{1 - \frac{c^2}{\alpha_2^2}},$$
$$a_{3i} = k(\mathrm{i}\mu_{15} - \mu_{35} n_i s_i - \mu_{55} s_i + \mathrm{i}\mu_{55} n_i) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{35} = 2\mathrm{i}k\mu_2 p_1,$$
$$a_{36} = 2\mathrm{i}k\mu_2 p_2,$$
$$a_{37} = k^2\mu_2\left(1 + \bar{b}^2\right),$$
$$a_{4i} = k(\mathrm{i}\mu_{13} - \mu_{33} n_i s_i - \mu_{35} s_i + \mathrm{i}\mu_{35} n_i) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{45} = \lambda_2 k^2 - \left(\lambda_2 + 2\mu_2\right) p_1^2,$$
$$a_{46} = \lambda_2 k^2 - \left(\lambda_2 + 2\mu_2\right) p_2^2,$$
$$a_{47} = 2\mathrm{i}\mu_2 \bar{b} k^2,$$
$$a_{5i} = k e^{k s_i h}(\mathrm{i}\mu_{15} - \mu_{35} n_i s_i - \mu_{55} s_i + \mathrm{i}\mu_{55} n_i) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{6i} = k e^{k s_i h}(\mathrm{i}\mu_{13} - \mu_{33} n_i s_i - \mu_{35} s_i + \mathrm{i}\mu_{35} n_i) \quad \text{for } i = 1, 2, 3, \text{ and } 4.$$

Appendix II

$$\chi_1 = P\mu_{33} - 2\mu_{33}\mu_{55},$$
$$\chi_2 = \mu_{13}^2 - \mu_{11}\mu_{33} + 2\mu_{13}\mu_{55} - \frac{c^2 P\rho_1}{2} + c^2\mu_{33}\rho_1 + c^2\mu_{55}\rho_1,$$
$$\chi_3 = -2\mu_{13}^2 + 2\mu_{11}\mu_{33} - 4\mu_{13}\mu_{55} + c^2 P\rho_1 - 2c^2\mu_{33}\rho_1 - 2c^2\mu_{55}\rho_1,$$
$$\chi_4 = -P\mu_{11} - 2\mu_{11}\mu_{15} + c^2 P\rho_1 + 2c^2\mu_{11}\rho_1 + 2c^2\mu_{55}\rho_1 - 2c^4\rho_1^2,$$
$$s_{11} = \sqrt{\frac{1}{\chi_1}\left(\chi_2 - \frac{\sqrt{\chi_3^2 - 4\chi_1\chi_4}}{2}\right)},$$
$$s_{22} = -\sqrt{\frac{1}{\chi_1}\left(\chi_2 - \frac{\sqrt{\chi_3^2 - 4\chi_1\chi_4}}{2}\right)},$$
$$s_{33} = \sqrt{\frac{1}{\chi_1}\left(\chi_2 + \frac{\sqrt{\chi_3^2 - 4\chi_1\chi_4}}{2}\right)},$$
$$s_{44} = -\sqrt{\frac{1}{\chi_1}\left(\chi_2 + \frac{\sqrt{\chi_3^2 - 4\chi_1\chi_4}}{2}\right)},$$

$$n_{ii} = \frac{\left(\mu_{55}-\frac{P}{2}\right)s_{ii}^2+\left(\rho_1 c^2-\mu_{11}\right)}{\mathrm{i}\left(\mu_{13}+\mu_{55}+\frac{P}{2}\right)s_{ii}} \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{3ii} = k(-\mu_{55}s_{ii} + \mathrm{i}\mu_{55}n_{ii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{4ii} = k(\mathrm{i}\mu_{13} - \mu_{33}n_{ii}s_{ii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{5ii} = ke^{ks_{ii}h}(-\mu_{55}s_{ii} + \mathrm{i}\mu_{55}n_{ii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{6ii} = ke^{ks_{ii}h}(\mathrm{i}\mu_{13} - \mu_{33}n_{ii}s_{ii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$s_{111} = \sqrt{\frac{-P-2\mu_1+2c^2\rho_1}{P-2\mu_1}},$$
$$s_{222} = -\sqrt{\frac{-P-2\mu_1+2c^2\rho_1}{P-2\mu_1}},$$
$$s_{333} = \sqrt{1-\frac{\rho_1 c^2}{\lambda_1+2\mu_1}},$$
$$s_{444} = -\sqrt{1-\frac{\rho_1 c^2}{\lambda_1+2\mu_1}},$$
$$n_{iii} = \frac{\left(\mu_1-\frac{P}{2}\right)s_{iii}^2+\left(\rho_1 c^2-(\lambda_1+2\mu_1)\right)}{\mathrm{i}\left(\lambda_1+\mu_1+\frac{P}{2}\right)s_{iii}} \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{3iii} = -k\mu_1(s_{iii} - \mathrm{i}n_{iii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{4iii} = k(\mathrm{i}\lambda_1 - (\lambda_1 + 2\mu_1)n_{iii}s_{iii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{5iii} = -k\mu_1 e^{ks_{iii}h}(s_{iii} - \mathrm{i}n_{iii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{6iii} = ke^{ks_{iii}h}(\mathrm{i}\lambda_1 - (\lambda_1 + 2\mu_1)n_{iii}s_{iii}) \quad \text{for } i = 1, 2, 3, \text{ and } 4.$$

Appendix III

$$a_0' = -\mu_{35}^2 + \mu_{33}\mu_{55},$$
$$a_1' = -2\mathrm{i}\mu_{15}\mu_{33} + 2\mathrm{i}\mu_{13}\mu_{35},$$
$$a_2' = \mu_{13}^2 - \mu_{11}\mu_{33} - 2\mu_{15}\mu_{35} + 2\mu_{13}\mu_{55} + c^2(\mu_{13} + \mu_{55})\rho_1,$$
$$a_3' = -2\mathrm{i}\mu_{13}\mu_{15} + 2\mathrm{i}\mu_{11}\mu_{35} - 2\mathrm{i}c^2\mu_{15}\rho_1 - 2\mathrm{i}c^2\mu_{35}\rho_1,$$
$$a_4' = -\mu_{15}^2 + \mu_{11}\mu_{55} - c^2\mu_{11}\rho_1 - c^2\mu_{55}\rho_1 + c^4\rho_1^2,$$
$s_i'(i = 1, \ldots, 4)$ are the roots of the equation: $a_0's^4 + a_1's^3 + a_2's^2 + a_3's + a_4' = 0,$
$$n_i' = -\frac{\left[\mu_{55}s_i'^2 - 2\mathrm{i}\mu_{15}s_i' + \left(\rho_1 c^2 - \mu_{11}\right)\right]}{\left[\mu_{35}s_i'^2 - \mathrm{i}(\mu_{13}+\mu_{55})s_i' - \mu_{15}\right]} \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$m_1' = \frac{c^2-\alpha_1^2+\alpha_2^2\bar{a}^2}{\mathrm{i}\bar{a}(\alpha_1^2-\alpha_2^2)},$$
$$m_2' = \frac{c^2-\alpha_1^2+\alpha_2^2\bar{b}^2}{\mathrm{i}\bar{b}(\alpha_1^2-\alpha_2^2)},$$
$$a_{3i}' = \mathrm{i}\mu_{15} - \mu_{35}n_i's_i' - \mu_{55}s_i' + \mathrm{i}\mu_{55}n_i' \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{35}' = -\mu_2(\mathrm{i}m_1' - \bar{a}),$$
$$a_{36}' = -\mu_2(\mathrm{i}m_2' - \bar{b}),$$
$$a_{4i}' = \mathrm{i}\mu_{13} - \mu_{33}n_i's_i' - \mu_{35}s_i' + \mathrm{i}\mu_{35}n_i' \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a_{45}' = -\left(\mathrm{i}\lambda_2 - (\lambda_2 + 2\mu_2)m_1'\bar{a}\right),$$
$$a_{46}' = -\left(\mathrm{i}\lambda_2 - (\lambda_2 + 2\mu_2)m_2'\bar{b}\right),$$

$$a'_{5i} = e^{ks'_i h}(\mathrm{i}\mu_{15} - \mu_{35}n'_i s'_i - \mu_{55}s'_i + \mathrm{i}\mu_{55}n'_i) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a'_{6i} = e^{ks'_i h}(\mathrm{i}\mu_{13} - \mu_{33}n'_i s'_i - \mu_{35}s'_i + \mathrm{i}\mu_{35}n'_i) \quad \text{for } i = 1, 2, 3, \text{ and } 4,$$
$$a'_{355} = \mathrm{i}m'_1 - \bar{a},$$
$$a'_{366} = \mathrm{i}m'_2 - \bar{b},$$
$$a'_{455} = \mathrm{i}\lambda_2 - (\lambda_2 + 2\mu_2)m'_1\bar{a},$$
$$a'_{466} = \mathrm{i}\lambda_2 - (\lambda_2 + 2\mu_2)m'_2\bar{b}.$$

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